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ONE-DIMENSIONAL FLOWS AND ISOMAGNETIC SHOCK WAVES IN A MAGNETIZED CONDUCTING MEDIUM

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§1. Consider the one-dimensional flow of an ideally conducting gas containing no space charge, which moves in a current tube in an electromagnetic field; the gas is inviscid and of zero thermal conductivity. For simplicity, we assume that the electric field **E** and magnetic field **H** are mutually perpendicular and lie in a plane perpendicular to the direction of motion. The equations for the flow take the following form [1, 2]:

$$\rho u = m = \text{const}; \tag{1.1}$$

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = \frac{1}{c} jB - \frac{1}{4\pi} \int_0^H \left(\mu - \rho\left(\frac{\partial\mu}{\partial\rho}\right)_{T,H}\right) H dH + \frac{1}{4\pi} B \frac{dH}{dx}; \tag{1.2}$$

$$\rho u \left(c_p \frac{dT}{dx} + u \frac{du}{dx}\right) + \rho u \frac{d}{dx}\left[\frac{1}{4\pi\rho} \int_0^H \left(T\left(\frac{\partial\mu}{\partial T}\right)_{\rho,H} - \rho\left(\frac{\partial\mu}{\partial\rho}\right)_{T,H}\right) H dH\right] = Ej; \tag{1.3}$$

$$\nu_m dH/dx = uB - cE,\ p = R\rho T,\ B = \mu H,\ j = \sigma(E - uB/c),\quad dE/dx = 0, \tag{1.4}$$

where the symbols are those commonly employed, with $\nu_m = c^2/4\pi\sigma$ the magnetic viscosity; the magnetic susceptibility is defined by Langevin's formula [3]:

$$\mu = 1 + (4\pi m_H \rho/MH)(\text{cth}\psi - 1/\psi),\ \psi = m_H H/kT, \tag{1.5}$$

where m_H and M are the magnetic moment and mass of one molecule of the perfect gas. Then (1.5) gives (1.2) and (1.3) the form

$$\rho u du/dx + dp/dx = (1/c)\sigma(E - uB/c)B + (1/4\pi)(\mu - 1) HdH/dx; \tag{1.6}$$

$$\rho u(c_p dT/dx + udu/dx) - \rho u(d/dx)[(kT/M)(\psi \text{cth}\psi - 1)] = \sigma(E - uB/c)E. \tag{1.7}$$

The method of [4] gives us expressions for the changes in speed and Mach number M along the current tube in terms of the flow parameters from (1.1), (1.4), (1.7), and the first two equations in (1.6):

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$$du/dx = -(\sigma H^2\mu^2/c^2p)(u-u_1)(u-u_3)/(\mathrm{M}^2-1); \tag{1.8}$$

$$d\mathrm{M}/dx = -(\sigma H^2\mu^2/c^2p)[(1+(\gamma-1)\mathrm{M}^2/2)/a_0(\mathrm{M}^2-1)](u-u_3)(u-u_2), \tag{1.9}$$

where

$$u_1 = [(\gamma-1)/\gamma]\{1+[(\gamma-1)/\gamma][1-\psi^2/\mathrm{sh}^2\psi]\}^{-1}(1-4\pi\rho u m_H/\mu McE);$$

$$u_2 = u_1\{\gamma\mathrm{M}^2+1+[(\gamma-1)/\gamma](\mathrm{M}^2-1)(1-\psi^2/\mathrm{sh}^2\psi)\}/(2+(\gamma-1)\mathrm{M}^2);$$

$$u_3 = cE/\mu H.$$

The following inequalities apply for a medium with magnetic susceptibility, as in magnetohydrodynamics: $u_2 < u_1 < u_3$ for $M < 1$ and $u_1 < u_2 < u_3$ for $M > 1$. The behavior of the integral curves from (1.8) and (1.9) in the Mu plane indicates that a supersonic flow can go over to a subsonic one and vice versa in such a medium at the points $u = u_3$ and $u = u_1$, with the supersonic–subsonic transition occurring at the maximum magnetic field in the current tube.

§2. Equations (1.2) and (1.3) with the last equation in (1.4) can be integrated to give

$$\rho u^2 + p + \frac{1}{4\pi}\int_0^H\left(\mu-\rho\left(\frac{\partial\mu}{\partial\rho}\right)_{T,H}\right)HdH = I; \tag{2.1}$$

$$\rho u\left(i_0+\frac{u^2}{2}+\frac{1}{4\pi\rho}\int_0^H\left(T\left(\frac{\partial\mu}{\partial T}\right)_{\rho,H}-\rho\left(\frac{\partial\mu}{\partial\rho}\right)_{T,H}\right)HdH+\frac{cEH}{4\pi}=\mathscr{E}; \tag{2.2}$$

$$i_0 = (p/\rho)\gamma/(\gamma-1),\ (c/4\pi)E = E_0. \tag{2.3}$$

The constants (m, I, $\mathscr{E}$, and E_0) are defined by specifying the fluxes of mass, momentum, energy, and electric field in a certain cross section, which can be taken as the initial one. Then (2.1) with (1.5) takes the form

$$p = I - H^2/8\pi - mu. \tag{2.4}$$

From (2.2), (2.4), (1.5), and the first equation in (2.3) we get the relation between the velocity and the magnetic field:

$$\begin{gathered} u^2-[2\gamma/(\gamma+1)](I-H^2/8\pi)u+[2(\gamma-1)/(\gamma+1)m](I-H^2/8\pi-\\ -mu)(\psi\operatorname{cth}\psi-1)+[2(\gamma-1)/(\gamma+1)](\mathscr{E}-E_0H)=0,\\ \psi=m_HH/kT=m_HmH/[Mu(I-H^2/8\pi-mH)]. \end{gathered} \tag{2.5}$$

The points corresponding to different values of x lie on the curve of (2.5) in the uH plane; the shape of this curve varies with the values for the constants. If $m_H H \ll kT$ (1.5) becomes $\mu = 1 + 4\pi m_H^2\rho/3kTM$, and when this expression is substituted into (2.5) and higher-order small quantities are deleted we get

$$u^2-[2\gamma/(\gamma+1)](I-H^2/8\pi)u+[2(\gamma-1)/m(\gamma+1)](\mathscr{E}-E_0H)=0. \tag{2.6}$$

This equation coincides with the corresponding equation in magnetohydrodynamics, namely, (1.9) of [5]; if $m_H H \gg kT$, the permeability is $\mu = 1 + 4\pi m_H\rho/HM$, and (2.5) becomes

$$u^2-[2\gamma/(\gamma+1)](I-H^2/8\pi)u+[2(\gamma-1)/m(\gamma+1)][\mathscr{E}-H(E_0-m_Hm/M)]=0. \tag{2.7}$$

This coincides with (1.9) of [3] if E_0 is replaced by $E_0 - m_Hm/M$ in the latter; comparison of (2.5) with (2.6) and (2.7) on the basis of the inequality $0 \le \coth\psi - 1/\psi < 1$ gives us that the curve represented by (2.5) in the uH plane, which is of interest only in the region $p > 0$, lies between the curves of (2.6) and (2.7). Further, the curve of (2.5) intersects straight lines parallel to the axes at not more than two points. Consequently, the curve of (2.5) has the form shown in Fig. 1 if E_0 is not too large [lines I-III correspond to equations (2.5)-(2.7)].

We integrate the first equation in (1.4) to get the radiation between the magnetic field and the coordinates:

$$x = \int \nu_m(u(H)\mu(H)H-4\pi E_0)^{-1}dH + \text{const.}$$

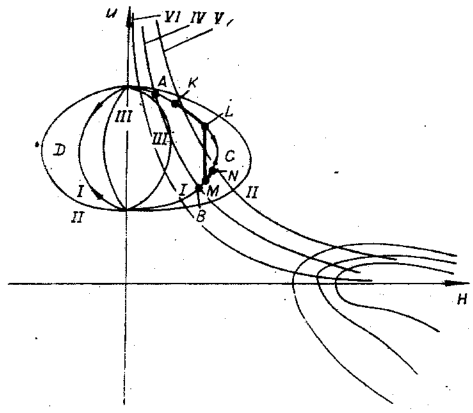

Fig. 1

The sections of the flow at which there is no current ($dH/dx = 0$) correspond to points on the $\mu Hu = 4\pi E_0$ curve; the latter equation may be used with (1.5) to give

$$[1 + (4\pi m_H H/uH)(\mathrm{cth}\psi - 1/\psi)]Hu = 4\pi E_0. \qquad (2.8)$$

The curve described by (2.8) lies between the hyperbolas

$$uH = 4\pi E_0, \; uH = 4\pi(E_0 - m_H m/M) \qquad (2.9)$$

that correspond to the cases $m_H H \ll kT$ and $m_H H \gg kT$; straight lines parallel to the axes intersect the curve of (2.8) at not more than one point each. Figure 1 shows the lines IV–VI corresponding to (2.8) and (2.9), with A and B denoting the points of intersection of curves I and IV, while C and D correspond to the points on curve I at which $du/dH = \infty$.

If the points corresponding to the values of u and H at some specified section of the current tube lie on line I above curve IV in the uH plane, then the magnetic field increases with x; if, on the other hand, this point lies below curve IV, the field decreases as x increases. The arrows in Fig. 1 indicate the direction of motion along the flow. From (1.8) and the first equation of (1.4) we get

$$du/dH = -H(u - u_1)/4\pi p(M^2 - 1). \qquad (2.10)$$

We see from this that point C corresponds to the point at which the flow speed is equal to the speed of sound; it is clear that the disposition of the curves shown in Fig. 1 makes it impossible for there to be a continuous transition through the speed of sound ($M = 1$) on moving along the flow for given m, I, $\mathscr{E}$, E_0, no matter what the initial values; a continuous transition through the speed of sound is possible only if points C and B coincide, which corresponds to obedience to the conditions $u = u_3$ and $M = 1$, simultaneously.

§3. If the parameters at some section of the tube are such that the flow cannot pass continuously through this section, then in certain cases we get flows containing shock waves in which the magnetic field is continuous, H = constant.

An example of such a flow is represented by the bold line KLMN. The parts KL and MN correspond to the continuous flow, while part LM corresponds to an isomagnetic step. The relationships at such steps take the form

$$\{p + m^2/\rho\} = 0; \qquad (3.1)$$

$$\{i_0 + u^2/2 - p(\psi\mathrm{cth}\psi - 1)\} = 0. \qquad (3.2)$$

These equations coincide with those for shock waves in gasdynamics for $m_H H \ll kT$.

If $m_H H \gg kT$, (3.2) may be put as

$$i_{02} + u_2^2/2 - m_H H\rho_2/M = i_{01} + u_1^2/2 - m_H H\rho_1/M. \qquad (3.3)$$

It follows from (3.1) and (3.3) that the adiabatic equation for an isomagnetic step takes the following form in the pV plane (V is specific volume):

$$p_2 = p_1(V_1\eta - V_2)/(\eta V_2 - V_1) - 2m_H H(V_2 - V_1)/MV_1V_2\ (\eta V_2 - V_1),\ \eta = (\gamma + 1)/(\gamma - 1). \tag{3.4}$$

The mass flux through the surface of discontinuity is given by the following formula, as in gasdynamics:

$$m^2 = (p_2 - p_1)/(V_1 - V_2).$$

It follows from (3.4) that the adiabatic curve in that case passes through the point p_1V_1 and has the same asymptotes as does the Hugoniot adiabatic and lies above the latter for $V_2 < V_1$, but below it for $V_2 > V_1$.

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CURRENT AND ENERGY AMPLIFICATION IN A PLANAR CUMULATIVE MAGNETIC GENERATOR WITH FLUX DIFFUSION

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§1. Magnetic-field compression in a conducting space (magnetic cumulation) increases the current and the magnetic-field energy; two cases are of interest here: 1) given an initial current I_0 and a load inductance L, select an initial circuit inductance L_0 such as to give the largest final current I; and 2) given the initial nergy U_0 and load L, select L_0 such as to obtain the largest energy U at the end.

Generators of the first type are used to produce very strong magnetic fields and may be called field generators; those of the second type are similarly called energy generators. The two types differ substantially in initial conditions: the initial current is preset in a field generator, and the energy is $U_0 \sim L_0$, while in an energy generator the initial energy is preset, and $I_0 \sim L_0^{-1/2}$. A field generator may be characterized via the current amplification factor

$$i = I/I_0 = (L_0/L)LI/L_0I_0 = \lambda\varphi, \tag{1.1}$$

where $\lambda = L_0/L$ represents the circuit change, and $\varphi = LI/L_0I_0$ is the proportion of the magnetic flux retained in the generator. An energy generator may be characterized by the energy amplification factor

$$\varepsilon = LI^2/L_0I_0^2 = \lambda\varphi^2. \tag{1.2}$$

§2. The quantity φ is a major characteristic of such a generator, as it is dependent on the design, and particularly on the conductivity σ of the material and the field-compression time. Also, the leakage of the flux

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